The Revealed Preference Approach to Demand

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Abstract

The standard approach to measuring demand responses and consumer preferences assumes particular parametric models for the consumer preferences and demand functions, and subsequently fits these models to observed data. In principle, the estimated demand models can then be used (i) to test consistency of the data with the theory of consumer behavior, (ii) to infer consumer preferences, and (iii) to predict the consumer’s response to, say, new prices following a policy reform. This chapter focuses on an alternative, nonparametric approach. More specifically, we review methods that tackle the above issues by merely starting from a minimal set of so-called revealed preference axioms. In contrast to the standard approach, this revealed preference approach avoids the use of parametric models for preferences or demand. The structure of the chapter is as follows. First, we introduce the basic concepts of the revealed preference approach to modeling consumer demand. Next, we consider issues like goodness-of-fit, power and measurement error, which are important in the context of empirical applications. Finally, we review a number of interesting extensions of the revealed preference approach, which deal with characteristics models, habit formation and the collective model.

Key words: Revealed preference, GARP, nonparametric analysis.

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1 Introduction

The other chapters in this volume start from a set of observations on consumer behavior with information on quantities and associated prices. This data set, then, serves to estimate a system of demand functions which relate all possible prices and total expenditures to the quantities purchased by the consumer. If these (continuous) demand functions add up, are homogeneous of degree zero and have a symmetric and negative semi-definite Slutsky matrix, then they can in principle be integrated into a rational preference ordering. This ordering information can then be used to analyze the impact of economic changes on the consumer’s well-being.

In this chapter, we take a different stance. The question posed here is whether it suffices to only make use of a finite set of observed price-quantity pairs to test whether observed behavior is rational and, subsequently, to conduct welfare analyses. The answer to this question is yes. More specifically, we will illustrate in this chapter how to test whether a fixed, finite set of observations on consumer behavior can be rationalized, i.e. each observed quantity bundle can be represented as maximizing a utility function subject to the relevant budget constraint. This test will be based on so-called ‘revealed preference’ theory, of which the origin goes back to Samuelson (1938, 1948). As we will further illustrate, once observed consumer behavior can be rationalized, it will in principle be possible to recover the underlying rational preferences and to conduct the type of welfare analyses discussed in the other chapters of this volume.

In this respect, a rightful question is why one would be interested in testing and recovering rational preferences based on revealed preference theory. After all, the theory and practical applicability of standard demand analysis is well established, as the other chapters have illustrated. Still, standard demand analyses suffer from a major deficiency: they are based on a particular functional specification for the demand system. This specification goes beyond the pure economic theory. From a testing perspective, this implies that one does not merely test rationality, but also the rather ad-hoc functional specification and other assumptions needed to go from the theory to the data. Consequently, a rejection of Slutsky symmetry may be either due to misspecification or to the fact that there does not exist any preference ordering that can rationalize observed consumer behavior. A major advantage of the revealed preference approach is that it allows testing rationality and recovering the underlying preference structure without making any assumptions about the

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1Early contributors to revealed preference theory like Samuelson (1938, 1948) or Houthakker (1950) still assumed the observability of the demand system, which gives a complete description of what would be chosen at any possible price vector and total expenditures level. Afriat (1967) lies at the origin of testing rationality and recovering a rational preference ordering with only a fixed set of price-quantity observations. Afriat’s work has been made more accessible by Diewert (1973) and Varian (1982); this last author also brought the theory to the data. The revealed preference theory summarized in this chapter is strongly embedded in the Afriat-Varian tradition. See Pollak (1990) and Varian (2006) for some background on the development of revealed preference theory and alternative revealed preferences axioms that are not explicitly considered in this chapter (such as the weak and strong axioms of revealed preference).
functional specification of demand or preferences. It only makes use of observed consumer behavior (in the form of a finite set of price-quantity pairs) to conduct demand analyses.

Of course, we should also stress that this revealed preference approach is subject to some criticisms. The most important one seems that it does not always provide precise predictions of the consumer's behavior in new economic environments or evaluations of the impact of economic reforms. Still, recent developments in revealed preference theory take away a great deal of these criticisms and generally allow conducting accurate welfare analyses. See in particular the methodological advances proposed by Blundell, Browning and Crawford (2003, 2008; see also Blundell, 2005), which will be discussed below.

One could also adopt a more pragmatic view in which the standard (parametric) approach is reconciled with the revealed preference approach. More concretely, a revealed preference analysis can serve as a pre-test for econometric analyses of demand. If a given set of price-quantity pairs passes a revealed preference test, then one can be sure that there exists a rational preference ordering that generates these choices. If a particular functional specification for the demand system applied to the same data rejects Slutsky symmetry, then one can infer from this that rationality as such is not rejected but rather the additional assumptions to bring the theory to the data.

In this respect, it is worth noting that there also exist revealed preference axioms that imply specific restrictions on the preferences, which in turn correspond to specific restrictions on the functional specification used for representing these preferences. For example, one can test whether observed consumer behavior can be rationalized in terms of homothetic or weakly separable preferences. Given the introductory nature of this chapter, we will not consider such tests. See, for example, Varian (1983) for a detailed discussion.

The outline of the rest of this chapter is as follows. In the next section, we will focus on the basic model for rational consumer behavior, which can be summarized in terms of Varian’s (1982) Generalized Axiom of Revealed Preference (GARP). We will discuss and illustrate both the testing of rationality and the recoverability of structural information needed to conduct welfare analyses. In Section 3, we will deal with specific empirical issues like goodness-of-fit and power of the basic model. We will also discuss how to integrate measurement error. Section 4 will devote attention to some recent extensions of the basic model, like the characteristics model, the habit formation model, and the collective model that explicitly recognizes the multi-person nature of multi-person households.

2 The basic model: GARP

2.1 Testing

Suppose that we observe a finite set of \( T \) price-quantity pairs. Let us denote the vectors of prices and quantities associated with observation \( t \) by \( \mathbf{p}_t \) and \( \mathbf{q}_t \) respectively, where \( \mathbf{p}_t \in \mathbb{R}_+^N \) and \( \mathbf{q}_t \in \mathbb{R}_+^N \). The data set \( S = \{ (\mathbf{p}_t; \mathbf{q}_t) , t = 1, \ldots, T \} \) represents the set of observations. Standard data sets with observed consumer behavior usually do not only
contain price and quantity information, but also information about the demographic composition of a household or other taste shifter variables like age or education level. Without losing generality, we will assume that the set $S$ refers to a particular household with given demographic and taste shifter variables in what follows.

Let us now define the rationality concept that we have in mind. Rationality basically implies that the data set under study could have been generated by a neoclassical utility maximizing consumer who is faced with a budget constraint:

**Definition 1 (rationality)** Let $S = \{(p_t; q_t); t = 1, ..., T\}$ be a set of observations. A utility function $U$ provides a rationalization of $S$ if for each observation $t$ we have $U(q_t) \geq U(q)$ for all $q$ with $p_t q \leq p_t q_t$.

The only condition imposed on this utility function is that it is locally non-satiated. As argued by Varian (1982), local non-satiation avoids trivial rationalizations of the data set: without this additional assumption, any observed household consumption behavior could be rationalized by a constant utility function $U(q_t) = \alpha$ for all $t \in \mathbb{R}$.

**Definition 2 (local non-satiation)** A utility function $U$ satisfies local non-satiation if the following holds. Suppose quantities $q_r$. Then for any $\epsilon > 0$ there exist quantities $q$ with $\|q - q_r\| < \epsilon$ such that $U(q) > U(q_r)$.

A core result in the revealed preference approach to demand is that there exists a locally non-satiated utility function that provides a rationalization of the set of observations $S$ if and only if the data satisfy the Generalized Axiom of Revealed Preference (GARP).

**Definition 3 (GARP)** Let $S = \{(p_t; q_t); t = 1, ..., T\}$ be a set of observations. The set $S$ satisfies the GARP if there exist relations $R_0, R$ that meet:

(i) if $p'_s q_s \geq p'_t q_t$ then $q_s R_0 q_t$;

(ii) if $q_s R_0 q_u, q_u R_0 q_v, ..., q_z R_0 q_t$ for some (possibly empty) sequence $(u, v, ..., z)$ then $q_s R q_t$;

(iii) if $q_s R q_t$ then $p'_s q_s \leq p'_t q_t$.

In words, the bundle of quantities $q_s$ is directly revealed preferred over the bundle $q_t$ (i.e. $q_s R_0 q_t$) if $q_s$ were chosen when $q_t$ were equally attainable (i.e. $p'_s q_s \geq p'_t q_t$); see condition (i). Next, the revealed preference relation $R$ exploits transitivity of preferences; see condition (ii). Finally, condition (iii) imposes that the bundle of quantities $q_t$ cannot be more expensive than revealed preferred quantities $q_s$.

As indicated above, any set $S$ of price-quantity pairs can be rationalized by a locally non-satiated utility function if and only if these price-quantity pairs satisfy GARP. This remarkable result is formalized and extended in the following theorem (Varian, 1982; based on Afriat, 1967):
Theorem 1 (Afriat Theorem) Let $S = \{(p_t; q_t); t = 1, \ldots, T\}$ be a set of observations. The following statements are equivalent:

(i) There exists a utility function $U$ that satisfies local non-satiation and that provides a rationalization of $S$;

(ii) The set $S$ satisfies GARP;

(iii) For all $t, r \in \{1, \ldots, T\}$, there exist numbers $U_t, \lambda_t \in \mathbb{R}_{++}$ that meet the Afriat inequalities

$$U_r - U_t \leq \lambda_t p_t(q_r - q_t);$$

(iv) There exists a continuous, monotonically increasing and concave utility function $U$ that satisfies local non-satiation and that provides a rationalization of $S$.

In this result, condition (ii) implies that data consistency with GARP is necessary and sufficient for a rationalization of the data. Condition (iii) provides an equivalent characterization in terms of the so-called Afriat inequalities. These inequalities allow an explicit construction of the utility levels and the marginal utility of income associated with each observation $t$ (i.e., utility level $U_t$ and marginal utility of income $\lambda_t$ for observed quantities $q_t$). Finally, condition (iv) states that, if there exists a utility function that provides a rationalization of the set $S$, then there exists a continuous, monotone and concave utility function that provides such a rationalization. This also implies that continuity, monotonicity and concavity of the data rationalizing utility function is non-testable for the basic model. In other words, if a utility function exists that rationalizes the data set $S$, then the data can in fact also be rationalized by a utility function with the nice properties of continuity, monotonicity and concavity. A powerful result indeed.

Figures 1 and 2 illustrate GARP. Figure 1 corresponds to a data set $S$ with three observations ($T = 3$). The data set consists of price-quantity pairs associated with two goods ($N = 2$). The slopes of the lines through the three quantity bundles $q_1$, $q_2$ and $q_3$ indicate the corresponding relative prices. It is easily checked that $q_1 R_0 q_2$: when the consumer purchased the bundle $q_1$, the bundle $q_2$ was also affordable but not purchased. The consumer thus revealed her or his preference for bundle $q_1$ over bundle $q_2$. In a similar way, we can derive that $q_2 R_0 q_3$. Bundles $q_1$ and $q_3$ cannot be compared in a direct revealed preference sense: neither bundle is contained in the other bundle’s budget set. However, by making use of transitivity, we can conclude that $q_1 R q_3$.

The data set in Figure 1 can be rationalized in the sense of Definition 1. Indeed, each observed bundle is expenditure minimizing over the corresponding set of revealed preferred bundles and, thus, the data set $S$ is consistent with GARP. Firstly, the revealed preferred set of bundle $q_1$ consists of only bundle $q_1$ itself, which implies that the first observation is expenditure minimizing in a trivial way. Secondly, the revealed preferred set of bundle $q_2$ consists of bundles $q_1$ and $q_2$. Also this observation is expenditure minimizing over its revealed preferred set: bundle $q_1$ could not be afforded when the consumer purchased bundle $q_2$. Thirdly, the revealed preferred set of bundle $q_3$ consists of bundles $q_1, q_2$ and
\(q_3\). Once again, this third observation is expenditure minimizing over its revealed preferred set: bundles \(q_1\) and \(q_2\) were associated with strictly higher total expenditures when the consumer purchased bundle \(q_3\).

Figure 2 shows an alternative data set \(S\) that is not consistent with \(GARP\). Both bundles \(q_1\) and \(q_2\) are in each other’s revealed preferred set. More specifically, \(q_1 \, R_0 \, q_2\) and \(q_2 \, R_0 \, q_1\). However, neither of the bundles turns out to be expenditure minimizing with respect to its revealed preferred sets. For example, it is easily seen that we have \(q_2 \, R \, q_1\) while \(p'_1 q_1 > p'_1 q_2\), which violates condition (iii) in Definition 3.

Figures 1 and 2 indicate the general structure of a \(GARP\) test for a given data set \(S\). Basically, such a test proceeds in two steps. In the first step, one recovers the (direct) revealed preference relations \(R_0\) and \(R\) (see conditions (i) and (ii) in Definition 3). In the second step, one checks whether each observation is effectively expenditure minimizing compared to its revealed preferred set (see condition (iii) in Definition 3). From a practical perspective, the most difficult part in this two-step test pertains to the first step, i.e. efficient recovery of the relation \(R\) as the transitive closure of the relation \(R_0\). An easy to implement and efficient algorithm to compute this transitive closure is provided by Warshall (1962); see Varian (1982) for discussion. In the Appendix to this chapter, we provide a programme code that can be used for testing consistency with \(GARP\) of a given data set. As discussed in the introduction, this \(GARP\) test can also serve as a useful pre-test in the context of a standard parametric demand analysis.
2.2 Recoverability

We next turn to the recoverability issue. To set the stage, let us first draw the parallel with the typical parametric approach to this issue. Parametric recoverability/identifiability aims at recovering the structural model parameters of a pre-specified direct or indirect utility function, which represents unique preferences, from a set of demand (reduced form) parameters that are estimated. By contrast, from a revealed preference perspective, there are usually many types of preferences that rationalize data consistent with GARP. So the recoverability question that we have in mind focuses on identifying the set of preferences (or set of utility functions representing different preferences) that are consistent with a given data set. More specifically, the recoverability question basically aims at constructing inner and outer bounds for the indifference curves passing through an arbitrary, not necessarily observed, quantity bundle. This construction is primarily based on restrictions upon behavior imposed by the GARP condition. In what follows, we will restrict to sketching the basic intuition of the revealed preference approach to the recoverability issue; we refer the interested reader to Varian (1982) for more details.

We illustrate the approach by means of Figure 3. The figure shows a very simple data set $S$ with only one observation $q_1$ (relative prices are again represented by the slope of the line). Now consider a new, non-observed bundle of quantities $q$. Can we say something about which bundles would be revealed preferred to the bundle $q$, and which bundles are revealed worse than $q$? In other words, can we recover information on the indifference curves that pass through $q$? Actually, we can. Basically, the revealed preference approach allows
us to make robust statements about this, which hold true for all possible prices associated
with $q$ that make the bundles $q_1$ and $q$ consistent with $GARP$.

We first conceive an inner bound for the indifference curves through $q$. As shown by
Varian (1982), the set of all possible bundles that are revealed preferred to $q$ is equal to
the convex monotonic hull of all bundles under study that are revealed preferred to $q$. In
Figure 3 this set consists of $q_1$ and $q$ itself. Therefore, the set of all bundles that are revealed
preferred to $q$ is equal to the set $RP(q)$ that is represented by the dark shaded area in
Figure 3; independent of the prices that are used, all the bundles in this set are revealed
preferred to $q$ (either directly or indirectly via the bundle $q_1$). As such, the boundary of
this set $RP(q)$ provides an inner bound for all possible indifference curves passing through
$q$.

Let us now focus on the outer bound of these indifference curves. Here, we want to
c caracterize the bundles that are revealed worse to $q$ for all prices that make the bundles
$q_1$ and $q$ consistent with $GARP$. Given the data at hand, prices $p$ associated with $q$ cannot
imply a budget line that is steeper than the line passing through $q_1$ and $q$. Indeed, otherwise
a violation of $GARP$ would occur: $q_1 \succ R q$ while $p'q > p'q_1$. Consequently, only budget
lines with a slope smaller than the slope of the line passing through $q_1$ and $q$ are possible.
It is now easily checked that for all these prices, the bundle $q$ is directly revealed preferred
to all bundles in the light shaded area $RW(q)$. The boundary of this area that passes through $q$ is the outer bound of all possible indifference curves passing through $q$.

It is clear from Figure 3 that the inner and outer bounds that are recovered can be far
apart from each other. This may serve as an illustration of the critique that a revealed preference approach has little bite in practice: *in casu*, this means that indifference curves can be very different from each other and still be consistent with observed behavior. However, it is worth emphasizing that the results in Figure 3 pertain to a data set with only one observation. Generally, the inner and outer bounds will be closer together if more observations are available. This is illustrated by Figure 4. This figure has exactly the same interpretation as Figure 3, but with four observed bundles \((q_1, q_2, q_3, q_4)\) rather than one.

An issue that is closely related to recoverability concerns predicting consumer behavior in new situations, characterized by a non-observed budget constraint. Interestingly, it is possible to make such predictions by only using revealed preference theory. Figure 5 provides an illustration. The figure shows a data set \(S\) with two observations. Suppose now that the dashed line represents the budget line in a new, non-observed, situation. It is clear that all bundles that exhaust this budget are within the reach of the consumer. However, not all these bundles are consistent with \(GARP\). Actually, only the bundles on the bold line segment are consistent with \(GARP\). The other bundles on the dashed line generate inconsistencies with rationality because they are not expenditure minimizing with respect to their revealed preferred set (see condition (iii) in Definition 3). Once again, it is clear that the set of rational outcomes (i.e. the bold line segment in Figure 5) will generally shrink if more observations are available.

As a final note, we refer to recent research by Blundell, Browning and Crawford (2003,
2008), which shows that one can substantially enhance recovery and prediction results by combining revealed preference theory with the nonparametric estimation of Engel curves. In fact, this effectively addresses the above-mentioned concern that a revealed preference approach may have little bite in practical applications. We will return to this in the next section.

3 Empirical issues

3.1 Goodness-of-fit

The above discussion of testing GARP consistency may seem quite peculiar from a traditional parametric point of view. Specifically, the GARP test described above is a ‘sharp’ test. The test tells us whether a finite set of observations is exactly consistent with the hypothesis that a consumer maximizes a utility function subject to a given budget. However, as argued by Varian (1990), exact optimization is not a very interesting hypothesis. Rather, we want to know whether rationality as defined in Definition 1 provides a reasonable way to describe observed behavior. For most purposes, ‘nearly optimizing behavior’ is just as good as ‘optimizing’ behavior. In this respect, it makes sense to investigate how large observed violations of rationality are in terms of economic significance. An answer to this question is provided by so-called goodness-of-fit measures.

An interesting goodness-of-fit measure is the ‘improved violation index’ (or ‘efficiency index’) that was proposed by Varian (1993). It indicates the degree to which the data are
‘optimizing’ or consistent with *GARP*.

We refer to Varian (1993) and Cox (1997) for an in-depth formal discussion of Varian’s improved violation index and restrict to a graphical illustration in this chapter. Figure 6 shows a data set $S$ with two observed bundles $q_1$ and $q_2$ that is not consistent with *GARP* (compare with our discussion of Figure 2). The basic concept in Varian’s procedure is the ‘violation index’. For each observation, this index is the minimal expenditure, computed over the revealed preferred set, divided by the actual observed expenditure. As such, if the observation is consistent with *GARP* then the index equals 1. On the other hand, when it is inconsistent with *GARP*, which means that it is not expenditure minimizing over his revealed preferred set, then the index is smaller than 1.

With a slight abuse of notation, the proportions $0q_1’/0q_1$ and $0q_2’/0q_2$ are the violation index values associated with respectively observation 1 and observation 2. In the Varian (1993) terminology, both observations in Figure 6 are involved in a ‘revealed preference cycle’. Varian then proposes a procedure that identifies the minimal expenditure perturbations needed to ‘break’ this cycle. The central idea behind the procedure is that a cycle can often be eliminated by perturbing just one of the budget hyperplanes involved in the cycle; it is not necessary to shift the budgets of all the consumption bundles. Specifically, Varian’s procedure starts from the basic violation index to construct an improved violation index for each observation. This improved index captures the minimal budget perturbations associated with the respective consumption bundles to obtain consistency with *GARP*.

In our graphical example, it suffices to shift the budget hyperplane through $q_1$ by a (positive) factor that is strictly below the associated violation index. A test for optimizing behavior that is weaker than the original ‘sharp’ test then multiplies the original expenditure level of observation 1 by that factor, while leaving the expenditure level of observation 2 unaltered. It turns out that we cannot reject *GARP* for these newly constructed expenditure values: observation 1 is no longer strictly revealed preferred to observation 2 and is itself expenditure minimizing over its revealed preferred set (which includes both observations). Notice that $0q_1’/0q_1$ is closer to unity than $0q_2’/0q_2$, so that shifting the budget hyperplane through $q_1$ is less drastic than shifting the budget hyperplane through $q_2$.

Of course, in the general case with multiple observations, more than two bundles are often involved in a revealed preference cycle. For this case, Varian proposes an iterative algorithm for computing improved violation indices.

### 3.2 Power

A full empirical assessment should contain more than only a goodness-of-fit analysis. We believe it is important to additionally consider the power of the revealed preference tests.

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2Varian (1990; based on Afriat, 1973) originally focused on the concept ‘violation index’; the improved violation index is a refinement of this original concept. The programme code in the Appendix calculates the original violation index of the quantity bundles in a given data set, and takes the average of these index values as a simple, alternative, goodness-of-fit measure.
Indeed, favorable goodness-of-fit results, indicating few violations of optimizing behavior, have little meaning if the behavioral implications are hardly restrictive, i.e. optimizing behavior can hardly be rejected. This is quite an important concern since, as already indicated, a criticism on the revealed preference approach is that it may have little bite in practice. In what follows, we will consider two related questions. First, we consider the issue of power measurement. Subsequently, we discuss the possibility to improve the power of the revealed preference tests in practical applications.

As for the first question, we follow Bronars’ (1987) approach to the power measurement issue; see also Andreoni and Harbaugh (2006) for recent discussion of this issue. Essentially, given that the revealed preference tests check rationality of the observed consumption behavior, power measures quantify the probability of detecting irrational behavior. Bronars suggested to take Becker’s (1962) notion of irrational behavior as the alternative model. This model states that a consumer chooses quantity bundles randomly from her or his budget set such that the budget is exhausted. More specifically, Beckerian irrational behavior means that the consumer chooses quantity bundles from a uniform distribution across all bundles in the budget hyperplane. Bronars’ power measure then captures the probability of rejecting the null hypothesis of optimizing behavior in the case of such irrational behavior. As discussed by Bronars, this probability depends on the number of budget set intersections associated with the different consumption bundles in the data set under study. If there are no intersections at all, then irrational behavior can never be detected.

Concretely, the procedure to calculate the power of the GARP test starts with the
simulation of irrational/random behavior for each observation in the data set $S$. That is, for the different goods in the demand bundle, randomly drawn budget shares are obtained from a continuous uniform distribution. The generated budget shares are then multiplied by observed total expenditure and divided by the actual price of each commodity, to obtain a random quantity of each good. In a second step, consistency with GARP is checked for a data set consisting of the simulated quantity bundles and the observed prices. This procedure is repeated many times.

In Bronars (1987), the power measure is equal to the number of times a GARP violation is detected in the different randomly constructed data sets divided by the number of repetitions. This power measure is thus based on the entire sample in the sense that the measure reveals the probability that irrational behavior of at least one observation in the sample is detected. However, alternative measures are possible. In Cherchye and Vermeulen (2008), for example, the power is evaluated at the level of each individual observation. Their reasoning is that there is a stronger case for a model that has high power in many observations than for a model with high power in only a few observations.

Let us now consider the second question, i.e. improve the power of the revealed preference tests in practical applications. This concern is particularly relevant in applications where the revealed preference approach effectively has little power, which is often the case when using real-life data sets. As for this issue, a very fruitful approach has recently been proposed by Blundell, Browning and Crawford (2003, 2008). Their ‘sequential maximum power’ (SMP) approach combines revealed preference theory with the nonparametric estimation of consumer expansion paths (i.e. Engel curves). Like before, given the introductory nature of this chapter, we will restrict to graphically sketching the main intuition. The interested reader is referred to Blundell, Browning and Crawford (2003, 2008) and Blundell (2005) for in-depth formal discussions.

Figure 7, which is borrowed from Blundell, Browning and Crawford (2003), considers a data set with three expansion paths; these nonparametrically estimated expansion paths correspond to observed price regimes with correspondingly chosen quantity bundles $q_1$, $q_2$ and $q_3$. The SMP approach then starts from a pre-defined preference ordering of the observed quantity bundles. Given this ordering, the idea is to construct virtual bundles on the expansion paths of the given price regimes such that, for a given observation that is evaluated, the power of the revealed preference test is maximized. This obtains the ‘sequential maximum power’ path. In principle, depending on the chosen ordering of the observed quantity bundles, a lot of such paths can be constructed. If there is no obvious preference ordering to start from, then one can check robustness of the test results with respect to alternative orderings.

In Figure 7 we illustrate the procedure for an ordering that begins at the third observation and finishes at the first observation. The outlay for the first observation is $x_1 = p_1^0 q_1(x_1) = p_1^0 q_1$. Let us now consider the construction of the virtual quantity bundles $\tilde{q}_2$ and $\tilde{q}_3$. As for $\tilde{q}_2$, the SMP procedure sets the outlay of the second observation so that the first period’s quantity bundle is just affordable: $\tilde{x}_2 = p_2^0 q_1(x_1)$. Under the crucial
assumption that demand is normal, the quantity bundle \( \bar{q}_2 = q_2 (x_2) \) is the lowest point on the second period’s expansion path that is directly revealed preferred to \( q_1 (x_1) \). The construction of the virtual bundle \( \bar{q}_3 \) proceeds analogously: we set \( \bar{x}_3 = p'_0 q_2 (\bar{x}_2) \) to obtain the quantity bundle \( \bar{q}_3 = q_3 (\bar{x}_3) \), which is the lowest point on the third period’s expansion path that is directly revealed preferred to \( q_2 \). In sum, we have that \( \bar{q}_3 R_0 q_2 \) and \( q_1 \), which respects the pre-defined ordering of the observed quantity bundles. Obviously, this situation is associated with a GARP violation, since \( q_1 \) is not expenditure minimizing with respect to the bundle \( \bar{q}_3 \) that is in its revealed preferred set.

An important result shown by Blundell, Browning and Crawford (2003) is that the GARP test based on the SMP procedure has maximum power, in the following sense: if one cannot reject GARP for a given SMP finishing at some quantity bundle, then this conclusion extends to any other preference ordering that finishes at the same bundle and maintains the same preference ordering. We can illustrate this by the example in Figure 7. For this example, it is clear that there is no GARP violation in terms of the originally observed bundles \( q_1, q_2 \) and \( q_3 \) (instead of \( q_1, \bar{q}_2 \) and \( \bar{q}_3 \), which do obtain a violation). By contrast, any bundle on the budget line through \( \bar{q}_3 \) that is on the line segment between the vertical axis and the budget line associated with \( x_1 \) would generate a GARP violation. Generally, it can be verified that any other budget generated with the expansion path \( q_3 (x) \) is associated with a smaller line segment (on the correspondingly defined budget line) that generates a GARP violation, which implies a smaller probability of detecting such a violation (i.e. lower power).

We conclude that the SMP approach counters the criticism that revealed preference tests may be associated with low power. Figure 8 shows that the same approach can also be useful for obtaining more powerful recoverability results, which -recall- focus on the construction of inner and outer bounds for an indifference curve. The original sets \( RP(q) \) and \( RW(q) \) are the same as in Figure 3. The light shaded area shows the gain by using expansion paths: we obtain tighter inner and outer bounds for the indifference curve passing through \( q \), which implies that more precise statements can be made. Similar gains apply to the construction of true cost-of-living indices and the recovery of demand responses to price changes; see Blundell, Browning and Crawford (2003, 2008) for practical applications.

### 3.3 Measurement error

Up to now, we have been assuming that the quantity and price bundles in the data set \( S \) are perfectly observed. Still, it is possible to adapt the revealed preference framework to take measurement error into account; see Varian (1985). In the following, we focus on measurement error in the quantity data. Still, it is equally possible to account for measurement error in the price data; the treatment is analogous.

Let us denote the ‘true’ quantities by the vectors \( q_t = (q_{1,t}, ..., q_{N,t})' \). They can be different from the observed quantities \( \tilde{q}_t = (\tilde{q}_{1,t}, ..., \tilde{q}_{N,t})' \). To account for measurement error, the following relationship between true and observed quantities can be assumed:
Figure 7: Testing GARP with expansion paths

Figure 8: Inner and outer bounds for an indifference curve using expansion paths
\[ q_{n:t} = q_{n:t} + \eta_{n:t} \text{ for } n = 1, \ldots, N \text{ and } t = 1, \ldots, T, \]

with the error term \( \eta_{n:t} \) assumed to be an independently and identically distributed random variable drawn from \( N \left( 0, \sigma^2 \right) \), with \( \sigma^2 \) the variance of the measurement error. A statistical test for data consistency with the utility maximization hypothesis consists of computing the test statistic

\[ \sum_{n=1}^{N} \sum_{t=1}^{T} \frac{(\bar{q}_{n:t} - q_{n:t})^2}{\sigma^2}. \]  

Under the null hypothesis that the true data satisfy \( GARP \), the test statistic follows a Chi-squared distribution with \( NT \) degrees of freedom. As such, consistency with \( GARP \) would be rejected if this test statistic exceeded the critical value that corresponds to a specified significance level. However, this test statistic is not observable. Therefore, following Varian (1985), a lower bound on the above statistic can be calculated by means of the programme

\[ \min \sum_{n=1}^{N} \sum_{t=1}^{T} \frac{(\bar{q}_{n:t} - q_{n:t})^2}{\sigma^2} \]  

subject to the vectors \( \bar{q}_t = (\bar{q}_{1:t}, \ldots, \bar{q}_{N:t})' \) satisfying \( GARP \).

Under the null hypothesis, the ‘true’ data satisfy the constraint, which implies that the resulting function value of the minimization programme in (2) should be no larger than the test statistic (1). Consequently, if we reject the null hypothesis on the basis of a function value for (2), then we certainly reject the null hypothesis on the basis of the true test statistic.

In practice, an important difficulty concerns the specification of the variance \( \sigma^2 \). Varian (1985) discusses two alternative solutions. First, we can use estimates of the error variance derived from (parametric or nonparametric) fits of the data, or from knowledge about how accurately the variables were measured. Alternatively, one can calculate how big the variance needs to be in order the reject to the null hypothesis of \( GARP \), and compare this to prior opinions regarding the precision with which the data have been measured.

4 Some extensions of the basic model

In this section we explore three extensions of the revealed preference method to different models of optimizing behavior.

4.1 Characteristics model

The notion that consumers have preferences primarily over the characteristics of market goods\(^3\) has turned out to be an extremely fruitful one with applications in areas from

index numbers (Stone, 1956), quality measurement (Griliches, 1971), location decisions (Tinbergen, 1959), labour market allocations (Heckman and Scheinkman, 1987), finance (Markowitz, 1959) and oligopoly models (Berry, Levinsohn and Pakes, 1995). The consumer characteristics model posits that, rather than having preferences over market goods directly, agents have preferences over the characteristics or attributes that these goods embody. The transformation from a \( N \)-vector of market goods, \( q \), to a \( J \)-vector of characteristics, \( z \), can, in general, be described by any nonlinear function but in this section we simply follow the most widely analyzed form and assume that this mapping is linear.\(^4\) That is \( z = A'q \) where \( A \) is a \((N \times J)\) technology matrix recording the amounts of each of the characteristics present in one unit of each of the market goods. The matrix \( A \) is assumed to have full column rank (i.e. the characteristics are distinct). Note that this standard specification involves a dimension reduction \((J < N)\) and it is this which gives the characteristics model its bite. This is because it reduces the number of objects over which the consumer has preferences from the potentially very large set of market products \((N)\) to a smaller number of product attributes. If there were no dimension reduction involved in the characteristics model then the model would be much less useful empirically than it is, and also it would be nonparametrically indistinguishable from the standard preference-for-goods model (i.e. it would imply nothing beyond \( GARP \)). The consumer choice model for given prices \( p \) and outlay \( x \) is:

\[
\max_q v(z) \quad \text{subject to} \quad z = A'q \quad \text{and} \quad p'q \leq x, \quad q \geq 0.
\]

Our focus is on the circumstances under which data \( S = \{(p_t; q_t); t = 1, \ldots, T\} \) and a mapping \( A \) can be nonparametrically rationalized by this model. In this context the term ‘rationalize’ is defined as follows:

**Definition 4 (z-rationality)** A utility function \( v(z) \) z-rationalizes the data \( S = \{(p_t; q_t); t = 1, \ldots, T\} \) for the technology \( A \) if \( v(z_t) = v(A'q_t) \geq v(z) \) for all \( z \) such that \( z = A'q \) and \( p'q_t \geq p'q \).

This states that a utility function rationalizes observed choices if it assigns an equal or higher value to those bundles of characteristics which the consumer chooses, than it does to those alternative bundles of characteristics which could have feasibly been produced from affordable bundles of market goods. If a utility function z-rationalizes the data, this means that were it used in the consumer’s maximization problem set out above, then it would generate exactly the observed data \( S = \{(p_t; q_t); t = 1, \ldots, T\} \) for the posited technology \( A \). Clearly z-rationalization for any \( A \) matrix implies rationalization by the preference-for-goods model (Definition 1).

For a good which is purchased, the first order condition from the linear characteristics model gives the following characterization of its price as a weighted sum of the shadow

---

\(^4\)See Blow, Browning and Crawford (2008) for the conditions for the nonlinear version of the model and further discussion of the various issues raised below.
prices of its characteristics:
\[ p^k_t = a_k \pi_t = \sum_{j=1}^{J} a_{kj} \pi_{jt} \]  
(3)

where \( a_k \) denotes the \( k \)th row of \( A \) and:

\[ \pi_{jt} = (\lambda_t)^{-1} v_j (z_t) . \]

Thus the shadow price of a characteristic is defined as its marginal utility normalized by the marginal utility of total expenditure \((\lambda_t)\) (see Gorman, 1956, equation (5)). That the market price of a good that is bought can be viewed as a linear combination of the underlying shadow prices is the most important feature of characteristics models. If good \( k \) is not bought then we have the inequality:

\[ p^k_t \geq a_k \pi_t \]  
(4)

so that the market price is too high relative to the subjective valuation of the embodied attributes.

The next proposition gives the necessary and sufficient conditions for the characteristics model (see Blow, Browning and Crawford, 2008):

**Proposition 1** The following statements are equivalent.
(P) there exists a utility function \( v(z) \) which is non-satiated, continuous and concave in characteristics which rationalizes the data \( S = \{(p_t; q_t) ; t = 1,...,T\} \) for given \( A \).

(A) there exist numbers \( \{V_t, \lambda_t > 0\}_{t=1,...,T} \) and vectors \( \{\pi_t\}_{t=1,...,T} \) such that

\[
\begin{align*}
V_s & \leq V_t + \lambda_t \pi^t_s (A'q_s - A'q_t), \quad \forall s, t \\
p^k_t & \geq a_k \pi_t, \quad \forall k, t \\
p^k_t & = a_k \pi_t \text{ if } q^k_t > 0, \quad \forall k, t
\end{align*}
\]

(A1) (A2) (A3)

(L) there exist numbers \( \{U_t, \rho_t \geq 1,\}_{t=1,...,T} \) and vectors \( \{\sigma_t\}_{t=1,...,T} \) such that

\[
\begin{align*}
U_s & \leq U_t + \sigma^t_s (A'q_s - A'q_t), \quad \forall s, t \\
\rho_t p^k_t & \geq a_k \sigma_t, \quad \forall k, t \\
\rho_t p^k_t & = a_k \sigma_t \text{ if } q^k_t > 0, \quad \forall k, t
\end{align*}
\]

(L1) (L2) (L3)

(G) the data \( \{\pi_t, A'q_t\}_{t=1,...,T} \) pass GARP for some choice of \( \pi_t \) such that (A2) and (A3) are satisfied.

One important feature of these conditions is that they do not impose that the shadow prices (the \( \pi^t_j \)'s in (A)) are non-negative; that is, agents may have a negative valuation for some characteristics. Of course, some of the shadow prices must be positive otherwise condition (A3) could not hold. Nevertheless this feature of the characteristics model allows for the
possibility that market goods may represent a bundling of characteristics not all of which are individually desirable. Conditions \((A2)\) and \((A3)\) impose the linear pricing condition \((3)\). Conditions \((A)\) and \((G)\) are the characteristics model analogues of the conditions in Afriat’s Theorem. However, both present practical difficulties for testing. Condition \((G)\) requires that we first find the shadow prices in order to implement a GARP test and there is no known general algorithm to do this in a finite number of steps.\(^5\) Condition \((A)\) involves both non-linear functions of unknowns (the \(\lambda_t \pi'_t\) terms) and strict inequality constraints on unknowns \((\lambda_t > 0)\). To overcome these problems in implementing the test for rationalization we have derived condition \((L)\). This condition is in the form of the restrictions in the first step of a linear programming problem. Consequently we can employ standard linear programming techniques to find, in a finite number of steps, whether there exists a feasible set of unknowns which satisfy these constraints (the first step of the simplex method).

Thus far we have assumed that the researcher always observes the price of all goods, even of those goods that the consumer does not buy in a particular period. For some data sets this is not the case and we only observe prices the agent faced when we observe a product being bought. If so then we face a serious missing data problem. One solution may be to impute the missing prices in some way and try to fill-in the missing observations.

The problem with any imputation scheme, however, is that we can never know how much the outcome of the test depends on the imputation. Instead of polluting the test in this manner it is possible instead to regard the missing prices as simply another set of unknowns (along with the Afriat numbers and shadow prices) and to ask whether values for them exist such that the constructed data satisfy the conditions. Since we can always implicitly set the prices of the goods which are not bought very high, this obviously makes it easier to satisfy the conditions and so the resulting test will be weaker in this sense. In practice this is very straightforward and simply involves dropping conditions \(L2\) and \(L3\) from the linear programming problem and replacing them with the restriction that

\[
\rho_t p_t^+ = A^+_t \sigma_t, \quad \forall t,
\]

where \(p_t^+\) indicates the prices of the goods bought in period \(t\) (that is, those for which \(q_t^k > 0\) and where, therefore, the corresponding prices are observed) and \(A^+_t\) denotes the corresponding submatrix of \(A\). Essentially, this involves just dropping the inequality restrictions on the goods for which the prices are unobserved. Since we are losing restrictions the test is weaker than it would be if all prices were known. If the new equality condition holds then we can take any set of implied \(\sigma_i\)’s and \(\rho_i\)’s and simply set:

\[
p_t^0 = \rho_t^{-1} A_t^0 \sigma_t,
\]

\(^5\)The computational problem is akin to that encountered in revealed preference tests of weak functional separability (see Varian, 1983).
where $A_0^t$ is the submatrix that is obtained by extracting from $A_t$ the rows of $A_t$ corresponding to market goods where demand in period $t$ is zero. This condition satisfies $(L3)$ in Proposition 1. In this case the resulting $p_0^t$ vectors have the interpretation of being *virtual* prices; that is, at these prices consumers are just on the verge of buying market goods that they did not buy in period $t$. If the $\sigma_t$’s and the $\rho_t$ are not unique, these values will not be uniquely determined either and we can only identify sets of virtual prices.

To conclude this section we note that the conditions we have established are also useful for testing *GARP*. If we define $A = I_K$ then the preference-for-goods model and the characteristics models are identical (the technology matrix simply maps from market products back to market products) and the test for the characteristics model is identical to the test of the standard model. However, in the presence of missing prices the linear programming condition for z-rationalization has a clear advantage over the *GARP* condition. As pointed out by Varian (1988), *GARP* tests are generally ruled out by missing price data because inner products such as $p_i^t q_s^t$ can involve missing prices; for example, if $p_i^k$ is missing because $q_i^k = 0$, but $q_i^s > 0$. However, using the modification to the linear program described above, the preferences-for-goods model can still be tested in this framework albeit that the test is weakened by the presence of missing prices.

### 4.2 Habit formation

*GARP* is a test of a static model of consumer behavior in which the consumer’s budget in each period is given. The *GARP* conditions then look for intertemporal stability of preferences in terms of their ability to rationalize within-period spending allocations across goods *given* the consumer’s budget and prices in each period. However, revealed preference methods can also be extended to dynamic models in which the allocation of spending between periods is considered. An early example of this is the paper by Browning (1989) which developed a revealed preference type test of the strong rational expectations hypothesis version of the life cycle model. In this section we consider the extension of these methods to habit formation models.

Models which allow for various kinds of habit formation have been used profitably to analyze a wide variety of both microeconomic and macroeconomic issues. Microeconomic applications have, for example, included Becker and Murphy’s (1988) classic study of the price-responsiveness of addictive activities, Meghir and Weber’s (1996) work on intertemporal nonseparabilities and liquidity constraints and the explanation of asset-pricing anomalies such as the equity premium puzzle (Abel, 1990, Campbell and Cochrane, 1999, Constantinides, 1990). Macro-orientated studies have used habit-formation models to improve the ability of business cycle models to explain movements in asset prices (Jermann, 1998, Boldrin et al., 2001), to investigate the idea that economic growth may cause savings rather than the

---

6 Varian (1988) establishes that if one price is not observed then there are no RP restrictions. Our context is different since we assume that a price is not observed in a particular period if and only if the good is not bought in that period; in this case there are testable RP restrictions (as shown).
other way around (Carroll et al., 2000) and to explain the finding that aggregate spending tends to have a gradual hump-shaped response to various shocks (Fuhrer, 2000).

Compared to the standard discounted utility model the principal feature of the habit-formation model is the relaxation of consumption independence. The implication of consumption independence in the standard discounted utility model is that tastes in one period are unaffected by consumption in another. This, in effect, is an argument against the time-separability of preferences in the discounted utility framework. Whilst Kubler (2004) shows that nonparametric testing of general nonseparable intertemporal choice models is not possible, the canonical habits model is rather special: it is additive and breaks intertemporal separability in a fairly specific manner. As a result the habits model is nonparametrically testable on the basis of observables using ideas akin to those from the rationing literature (Neary and Roberts, 1980 and Spinnewyn, 1981).

Suppose we have \( T \) observations indexed by \( t \) on a consumer’s demands over time \( q_t \) and the corresponding prices \( p_t \) and interest rate \( i_t \). Let the commodity vector be partitioned into a group of consumption goods \( q^c_t \) and a group of goods which are thought to be habit-forming \( q^a_t \) such that \( q_t = [q^c_t, q^a_t]^T \). To develop the main ideas without the loss of a great deal of generality, the discussion will initially focus on the simplest case in which the effects of lagged consumption of the addictive goods only persist for one period. The discussion of this extension (which is straightforward) is postponed until the end of this section.

The model of interest is

\[
\max_{q^c_t, q^a_t} \sum_{t=1}^{\infty} \beta^{t-1} u(q^c_t, q^a_t, q^a_{t-1}) \quad \text{subject to} \quad \sum_{t=1}^{\infty} \beta^{t-1} \left( \rho_t^c q^c_t + \rho_t^a q^a_t \right) = A_0 \quad \text{and} \quad q^a_0 = q^a,
\]

where \( \beta = 1/ (1 + \delta) \), \( \delta \in [0, \infty) \), is the consumer’s rate of time preference, \( \rho_t^b = p_t^b / \prod_{s=2}^{t} (1 + i_s) \), \( b = c, a \), denotes discounted prices, \( i_t \) the interest rate, \( A_0 \) is the present value of wealth and \( q^a_0 = q^a \) defines the level of the initial habits stock. It is assumed that the instantaneous utility (felicity) function \( u \) is non-satiated, continuous, concave and monotonic. Thus the habits model considered in this paper is what we take to be the “canonical” version of the intrinsic habits model considered by Ryder and Heal (1973), Boyer (1978, 1983), Spinnewyn (1981), Iannaccone (1986), Becker and Murphy (1988) and Becker, Grossman and Murphy (1994) \textit{inter alia}. As Frederick et al. (2002, p. 369) point out, although this kind of habit formation model is often said to induce a preference for an increasing consumption profile (the “hedonic treadmill”), in fact they are much more flexible and can also allow for preferences for decreasing or even non-monotonic consumption profiles. Which is the case

\[7\]

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7 Other versions of habits models have been put forward in the literature. These include models in which consumers are myopic (Pollak, 1970), discount rates that depend on prior consumption (Shi and Epstein, 1993) extrinsic (keeping-up-with-the-Joneses) habits models (Abel, 1990) and Campbell and Cochrane, (1999) and models in which instantaneous utility/felicity is S-shaped (Loewenstein and Prelec, 1992, Camerer and Loewenstein, 2004)).
depends on various factors such as the level of the initial habits stock and whether current consumption raises or lowers future utility - in other words whether the habit-forming good is good for you or not. Becker, Grossman and Murphy (1994, p. 398), for example, employ this model in their study of cigarette addiction and use it to allow for the fact that current consumption can reduce future utility. Consistency between the habits model and the data is defined as follows.

**Definition 5 (Habit rationality)** The time series of the interest rate, prices and quantities \( \{i_t, p_f^t, p_a^t; q_f^t, q_a^t\}_{t \in \{2, ..., T\}} \) satisfies the one-lag habits model if there exists a non-satiated, continuous, concave, monotonic (utility) function \( u(\cdot) \) and positive constants \( \lambda \) and \( \beta \) such that

\[
\beta^{t-1}D_{q_f^t}u(q_f^t, q_a^t, q_{a,t-1}^a) = \lambda \rho_t^f
\]

\[
\beta^{t-1}D_{q_a^t}u(q_f^t, q_a^t, q_{a,t-1}^a) + \beta^tD_{q_a^t}u(q_{a,t+1}, q_{a,t+1}, q_{a,t}^a) = \lambda \rho_t^a
\]

where \( \rho_t^i = p_t^i / \prod_{s=2}^{t} (1 + i_s) \) denotes discounted prices.

This says that the data are consistent with the theory if there exists a well-behaved instantaneous utility (felicity) function (defined over the consumption goods and the habit-forming goods plus the one-period lag of the habit-forming goods), the derivatives of which satisfy the first order conditions of optimizing behavior. If such a utility function exists, and we know what it is, then it means that we can simply plug it into the habits model, solve the model and precisely replicate the observed demand choices of the consumer. To put it another way, the theory and the data are consistent if there exists a well-behaved utility function which can provide perfect within-sample fit of the consumption/demand data.

From Definition 5 it is clear that the first order conditions for the consumption goods are identical to those of the standard perfect foresight model. Those for the habit-forming goods are a little more complex because current consumption affects future utility as well as current utility - in the case of a priori harmfully addictive goods the discounted effect of current consumption on next period’s utility is negative, but in general the model simply allows this term to be non-zero.\(^8\) Despite this complication this condition can be transformed into a form which is analogous to a no-habits model by defining suitable shadow discounted prices which account for these welfare effects\(^9\):

\[
\rho_t^{a,0} = \frac{\beta^{t-1}D_{q_f^t}u(q_f^t, q_a^t, q_{a,t-1}^a)}{\lambda}
\]

\[
\rho_t^{a,1} = \frac{\beta^{t-1}D_{q_a^t}u(q_f^t, q_a^t, q_{a,t-1}^a)}{\lambda}
\]

\(^8\)See Becker, Grossman and Murphy (1994, p. 398) for a discussion of this point.

\(^9\)As pointed out by Spinnewyn (1981). See also Neary and Roberts (1980).
Expression (6) is the shadow discounted price of current consumption and measures the discounted willingness-to-pay for current consumption of the habit-forming goods. Expression (7) is the shadow discounted price of past consumption and measures the discounted willingness-to-pay for past consumption of the habit-forming goods. It is worth noting that the shadow discounted price of current consumption can be interpreted as the (observed) discounted price adjusted to account for the future welfare effects of current decisions. That is, using Definition 5,

$$\rho^{a,0}_t = \rho_t^0 - \frac{\beta_t \sum_{c} u(q^c_{t+1}, q^a_{t+1}; q^c_t)}{\lambda}. \quad (8)$$

Given (6), (7) and (8) the habits model entails an intertemporal dependence between the shadow discounted prices:

$$\rho_t^0 = \rho^{a,0}_t + \rho_t^{a,1}. \quad (9)$$

The empirical/behavioral implications of the short memory habits model are therefore driven by: (i) links between the derivatives of discounted utility with respect to future and past consumption of the habit-forming goods and the (unobservable) shadow discounted prices, and (ii) intertemporal links between the (unobservable) shadow discounted prices and the (observable) discounted prices. The aim then, is to turn these insights into testable empirical conditions involving only observables. The following result can now be given (see Crawford, 2008):

**Proposition 2** The following statements are equivalent:

(T) The time series of the interest rate, prices and quantities \( \{i_t, p^c_t, p^a_t; q^c_t, q^a_t\}_{t=2}^{T} \) satisfies the one-lag habits model.

(R) There exist shadow discounted prices \( \{\rho^{a,r}_t\}_{t=2}^{T} \) and a positive constant \( \beta \) such that

\[
0 \leq \sum_{s,t \in \sigma} \pi'_s (x_t - x_s) \quad \forall \sigma \subseteq t \in \{2, ..., T\} \quad (R1)
\]

\[
0 = \rho^a_t - \rho^{a,0}_t - \rho^{a,1}_{t+1} \quad \forall \quad t, t + 1 \in \{2, ..., T\} \quad (R2)
\]

where \( x_t = [q^c_t, q^a_t, q^a_{t-1}]' \) and \( \pi_t = \frac{1}{\beta^{t-1}} \left[ \rho^a_t, \rho^{a,0}_t, \rho^{a,1}_t \right]' \).

This says that if one can find suitable shadow prices and a discount rate such that restrictions (R1) and (R2) hold, then the data are consistent with the theory and there does indeed exist a well-behaved utility function which gives perfect within-sample rationalization of the data. Conversely if such shadow discounted prices and a discount rate cannot be found then there does not exist any theory-consistent utility representation. Restriction (R1) is a cyclical monotonicity condition\(^{10}\) which is an implication of the concavity of the instantaneous utility function and the constant marginal utility of lifetime wealth. This condition involves

\(^{10}\text{Rockafellar, (1970, Theorem 24.8)}\)
the shadow discounted prices discussed above. Restriction (R2) is the intertemporal link between the shadow prices.

The empirical test is thus a question of searching for shadow price vectors and a discount rate which satisfies the restriction in (R). These restrictions are non-linear in unknowns and look forbidding but are, in fact, computationally quite straightforward. The important feature to note is that, conditional on the discount rate, the restrictions are linear. This means that, for any choice of discount rate, the existence or non-existence of feasible shadow prices can be readily determined in a finite number of steps using phase one of a (simplex method) linear programme. The issue is then simply one of conducting an arbitrarily fine one-dimensional grid search for the discount rate and running a linear programming problem at each node.

To end consider a more general model in which consumption of the habit-forming goods persists for $R$ periods\(^{11}\) the instantaneous utility function is given by

$$u(q_t, q_t^0, q_{t-1}^0, q_{t-2}^0, \ldots, q_{t-R}^0)$$

(10)

The definition of what it means for data to be consistent with the R-lag model and the corresponding necessary and sufficient conditions for theoretical consistency\(^{12}\) are both natural extensions of Definition 5 and Proposition 2. Once more the restrictions come in the form of a cyclical monotonicity condition and an intertemporal condition linking the shadow and spot prices of the habit-forming goods. However in this more general model the lag lengths involved in the consumption vectors are longer and the intertemporal links between shadow prices extend further. In other respects the restrictions are multi-period analogues of those in Proposition 2.

4.3 Collective model

The above discussed models did not really make explicit what is meant by ‘the consumer’. Is it an individual or could it be a household that consists of a number of different household members? This is not a harmless question. A growing body of evidence suggests that the hypothesis of utility maximization subject to a budget constraint (also known as the ‘unitary model’) is at odds with observed multi-person household behavior (examples are Fortin and Lacroix, 1997, Browning and Chiappori, 1998, Cherchye and Vermeulen, 2008, and Cherchye, De Rock and Vermeulen, 2008). From a theoretical perspective, this may be quite obvious: we already know for a long time that it is very difficult to aggregate individual rational preferences into a single rational preference ordering.

A recent alternative to the unitary model, the so-called ‘collective model’, has been proposed by Chiappori (1988, 1992). This model explicitly takes account of the fact that multi-person households consist of several individuals with their own rational preferences;

\(^{11}\)It is assumed that the number of lags is strictly fewer than the number of observations. If this is not the case then obviously the habits model is untestable/unrejectable.

\(^{12}\)See crawford (2008) for these results.
household decisions are then the Pareto efficient outcomes of a bargaining process. Browning and Chiappori (1998) provided a characterization of a general collective consumption model, which allows for public consumption and externalities inside the household. They take the minimalistic prior that the empirical analyst does not know which goods are characterized by public consumption and/or externalities. In what follows, we establish a revealed preference characterization of the same general collective consumption model. More specifically, by using revealed preference axioms, conditions can be derived that allow for testing whether observed household consumption behavior is collectively rational, without imposing any parametric structure on the intra-household allocation process and individual preferences (possibly characterized by public consumption and positive externalities).

We will focus on the case with two household members (but all results can be generalized to households of any size). Like before, we consider a set of observations \( S = \{(p_t; q_t); t = 1, ..., T\} \). To model externalities and public consumption, we consider ‘personalized quantities’ \( \hat{q}_t = (q_{1t}, q_{2t}, q_{ht}) \). These personalized quantities decompose each (observed) aggregate quantity bundle \( q_t \) into quantities \( q_{1t} \) and \( q_{2t} \) capturing the private consumption of each household member and quantities \( q_{ht} \) representing public consumption. Of course, the different components of \( \hat{q}_t \) must add up to the aggregate quantity bundle for each observation \( t \):

\[
q_t = q_{1t} + q_{2t} + q_{ht}.
\]

Each member \( m \) has a non-satiated utility function \( U^m \) that is non-decreasing in these personalized quantities, which effectively accounts for (positive) externalities and public consumption.

The collective model then regards the observed household consumption as the Pareto efficient outcome of a bargaining process between the two household members. We obtain Definition 6, which provides a collective version of the rationality concept in Definition 1. The Pareto weight \( \mu_t \) can be interpreted as the relative bargaining weight for the second household member; it represents the weight that is given to this member’s utility in the intra-household optimization process.

**Definition 6 (collective rationality)** Let \( S = \{(p_t; q_t); t = 1, ..., T\} \) be a set of observations. A pair of utility functions \( U^1 \) and \( U^2 \) provides a collective rationalization of \( S \) if for each observation \( t \) there exist feasible personalized quantities \( \hat{q}_t \) and \( \mu_t \in \mathbb{R}_+^+ \) such that

\[
U^1(\hat{q}_t) + \mu_t U^2(\hat{q}_t) \geq U^1(\hat{q}) + \mu_t U^2(\hat{q})
\]

for all \( \hat{q} = (q^1, q^2, q^h) \) with \( q^1, q^2, q^h \in \mathbb{R}_+^N \) and \( p_t'(q^1 + q^2 + q^h) \leq p_t'q_t \).

Cherchye, De Rock and Vermeulen (2007) established testable (necessary and sufficient) revealed preference conditions for such a collective rationalization of the data. In doing so, they adopted the minimalistic prior that the empirical analyst only observes the aggregate bundle \( q_t \) and not its intra-household allocation; such unobservability is often the case in practical applications.

25
The starting point of the revealed preference condition for collective rationality is that the ‘true’ member-specific preference relations are not observed, because only the aggregate household quantities \( (q_s) \) are observed and not the ‘true’ personalized quantities \( (q^1_s, q^2_s \text{ and } q^3_s) \). Given this, the condition focuses on so-called ‘hypothetical member-specific preference relations’. These relations essentially represent feasible specifications of the true individual preference relations in terms of a number of collective rationality conditions (i.e., conditions \((i) \) to \((v) \) in Proposition 3 below) defined on the observed (aggregate household) quantities and prices. The revealed preference condition for collectively rational consumption behavior then requires that there must exist at least one specification of the hypothetical member-specific preference relations that simultaneously meets all these collective rationality conditions. The condition is summarized in the following proposition:

**Proposition 3** Suppose that there exists a pair of utility functions \( U^1 \) and \( U^2 \) that provide a ‘collective rationalization’ of the set of observations \( S = \{ (p^t_s; q^t_s) : t = 1, ..., T \} \). Then there exist ‘hypothetical’ relations \( H^m_0 \), \( H^m \) for each member \( m \in \{1, 2 \} \) such that:

(i) if \( p'_s q_s \geq p'_s q_t \), then \( q_s H^1_0 q_t \) or \( q_s H^0_1 q_t \);

(ii) if \( q_s H^0_0 q_k, q_k H^0_0 q_l, ..., q_l H^0_0 q_t \) for some (possibly empty) sequence \((k, l, ..., z)\), then \( q_s H^m q_t \);

(iii) if \( p'_s q_s \geq p'_s q_t \) and \( q_t H^m q_s \), then \( q_s H^1 q_t \) (with \( l \neq m \));

(iv) if \( p'_s q_s \geq p'_s (q_{l1} + q_{l2}) \) and \( q_{l1} H^m q_s \), then \( q_s H^1 q_{l2} \) (with \( l \neq m \));

(v) \[
\begin{align*}
\text{a)} & \text{ if } q_s H^1 q_t \text{ and } q_s H^2 q_t, \text{ then } p'_t q_t \leq p'_s q_s \\
\text{b)} & \text{ if } q_{s1} H^1 q_t \text{ and } q_{s2} H^2 q_t, \text{ then } p'_t q_t \leq p'_s (q_{s1} + q_{s2})
\end{align*}
\]

This condition has a formally similar structure as the **GARP** condition in Definition 3. The essential difference is that Proposition 3 imposes restrictions in terms of ‘hypothetical’ member-specific preference relations \( H^0_0 \) and \( H^m \), while **GARP** specifies restrictions in terms ‘observable’ revealed preference relations \( R_0 \) and \( R \).

Condition \((i)\) applies to all situations with \( p'_s q_s \geq p'_s q_t \). This means that the quantity bundle \( q_t \) was equally obtainable under the prices \( p_s \) and the outlay \( p'_s q_s \) that correspond to the chosen bundle \( q_s \). In that case, Pareto efficiency requires that at least one household member must prefer the bundle \( q_s \) to the bundle \( q_t \). If we assume that member \( m \) prefers \( q_s \) to \( q_t \), then we specify \( q_s H^0_0 q_t \). Summarizing, the inequality \( p'_s q_s \geq p'_s q_t \) requires that we specify \( q_s H^0_0 q_t \) for at least one \( m \). Condition \((ii)\) uses that individual preferences are transitive.

The following conditions \((iii)\) to \((v)\) pertain to rationality across the household members. Condition \((iii)\) expresses that, if member 1 prefers some \( q_t \) over \( q_s \), and the quantity bundle \( q_t \) is not more expensive than \( q_s \), then the choice of \( q_s \) can be rationalized only if member 2 prefers \( q_s \) over \( q_t \). Indeed, if the last condition were not satisfied, then the bundle \( q_t \) (under the given prices \( p_s \) and outlay \( p'_s q_s \)) would imply a Pareto improvement over the chosen bundle \( q_s \).
Similarly, condition (iv) states that, if the quantity bundle \( q_s \) is more expensive than the (newly defined) bundle \( (q_{t_1} + q_{t_2}) \), while member 1 prefers \( q_{t_1} \) over \( q_s \), then the only possibility for rationalizing the choice of \( q_s \) is that member 2 prefers \( q_s \) over the remaining bundle \( q_{t_2} \). The interpretation in terms of Pareto efficiency is directly similar to the one for condition (iii).

Finally, condition (v) complements conditions (iii) and (iv); it defines upper expenditure bounds for each observation \( t \) that depend on the specification of the relations \( H^m \). Part a) of condition (v) states that if both members prefer \( q_s \) over \( q_t \), then the choice of \( q_t \) can be rationalized only if it is not more expensive than \( q_s \). Indeed, if this last condition were not met, then for the given prices \( p_t \) and outlay \( p_t q_t \) all members would be better off by buying the bundle \( q_s \) rather than the chosen bundle \( q_t \), which of course conflicts with Pareto efficiency. Part b) of condition (v) expresses a similar condition for the case where both members prefer a different quantity bundle \( q_{s_m} \) to \( q_t \). In that case, the choice of \( q_t \) can be rationalized only if it is not more expensive than the bundle \( (q_{s_1} + q_{s_2}) \).

To summarize, conditions (i) to (v) imply a necessary revealed preference condition for collectively rational household behavior that can be tested on the available aggregate (price and quantity) information. It can be shown that the condition is rejectable in a two-person setting as soon as there are three goods and three observations. Applications of this condition can be found in Cherchye, De Rock, Sabbe and Vermeulen (2008) and Cherchye, De Rock and Vermeulen (2008), who also discuss algorithms to test the condition in an efficient way.

5 Conclusion

This chapter focused on the revealed preference approach to demand. It reviewed methods to test alternative behavioral models. These tests all start from a minimal set of so-called revealed preference axioms that are directly applied to the raw price-quantity observations; this is a most important difference with the parametric methods discussed in the other chapters of this book, which typically use some ad-hoc functional specification of the demand system. As we have illustrated, the revealed preference approach allows us to test different models, and to recover important information on the structural model that underlies the observed demand behavior (e.g. information on indifference curves). Apart from testability and recoverability, we have also focused on empirical issues like goodness-of-fit, power and measurement error.

Appendix

This appendix contains excerpts from a programme that allows testing whether a given set of price-quantity pairs is consistent with GARP (see Section 2). It further calculates observation-specific and average violation indices (see Section 3). The programme code
below is written in Fortran, but its structure should be clear enough to translate it easily to another package.

**Calculation of total expenditures associated with each price-quantity combination**

```fortran
do 1 i=1,T
   do 2 j=1,T
      pq(i,j)=0
      do 3 k=1,N
         pq(i,j)=pq(i,j)+p(i,k)*q(j,k)
      3 continue
   2 continue
1 continue
```

**Construction of direct revealed preference relations**

```fortran
do 4 i=1,T
   do 5 j=1,T
      if(pq(i,i).ge.pq(i,j)) then
         m(i,j)=1
      else
         m(i,j)=0
      endif
   5 continue
4 continue
```

**Construction of revealed preference relations (Warshall’s algorithm)**

```fortran
do 6 k=1,T
   do 7 i=1,T
      do 8 j=1,T
         if((m(i,k).eq.0).or.((m(k,j).eq.0))) then
            goto 8
         endif
         m(i,j)=1
      8 continue
   7 continue
6 continue
```

**Check whether each observation is expenditure minimizing with respect to the bundles that are revealed preferred to this observation, by computing observation-specific violation**
indices. A violation index that equals 1 implies that the observation is expenditure mini-
mizing. The data set can be rationalized only if all observations have a violation index that
equals 1. The mean violation index is also calculated.

    do 9 j=1,T
       mincost=pq(j,j)
       do 10 i=1,T
          if(m(i,j).eq.1) then
             if(pq(j,i).le.mincost) then
                mincost=pq(j,i)
             endif
          endif
       10 continue
       viol(j)=mincost/pq(j,j)
    9 continue
    violmean=real(0)
    do 11 i=1,T
       violmean=violmean+viol(i)/real(T)
    11 continue

References

American Economic Review, 80, 38-42.


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